

ON LIMITS OF *ab initio* CALCULATIONS OF PAIRING GAP IN NUCLEI

E.E. Saperstein,^{1,*} M. Baldo,² U. Lombardo,³ S.S. Pankratov,⁴ and M.V. Zverev⁵

¹*Kurchatov Institute, 123182 Moscow, Russia.*

²*INFN, Sezione di Catania, 64 Via S.-Sofia, I-95123 Catania, Italy.*

³*INFN-LNS and University of Catania,*

44 Via S.-Sofia, I-95125 Catania, Italy.

⁴*Kurchatov Institute, 123182 Moscow,*

Russia; Moscow Institute of Physics and Technology, 123098 Moscow, Russia.

⁵*Kurchatov Institute, 123182 Moscow, Russia;*

Moscow Institute of Physics and Technology, 123098 Moscow, Russia.

A brief review of recent microscopic calculations of nuclear pairing gap is given. A semi-microscopic model is suggested in which the *ab-initio* effective pairing interaction is supplemented with a small phenomenological addendum. It involves a parameter which is universal for all medium and heavy nuclei. Calculations for several isotopic and isotonic chains of semi-magic nuclei confirm the relevance of the model.

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1. INTRODUCTION

Up to now, there is no consistent microscopic theory of nuclear matter. The well-known Brueckner theory [1] was the first very promising step in this direction but next steps are very complicated as one deals with the many-body problem without any small parameter. Why the idea to develop the *ab initio* theory of pairing in finite nuclei is not absolutely unreasonable, although a finite nucleus is much more complicated system than infinite nuclear matter? The point is that, for the pairing problem, some simplifications occur in finite nuclei. They originate from the surface nature of nuclear pairing [2]. If the pairing problem is

* Electronic address: saper@mbslab.kiae.ru

formulated in terms of an effective pairing interaction V_{eff}^p in a model space S_0 , this quantity turns out to be density dependent [3, 4] with strong dominance of the surface attraction. To be more definite, let us write down the simple local 2-parameter ansatz for V_{eff}^p within the Finite Fermi Systems (FFS) theory [5]:

$$V_{\text{eff}}^p(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3, \mathbf{r}_4) = C_0 \left[\gamma^{\text{ex}} + (\gamma^{\text{in}} - \gamma^{\text{ex}}) \frac{\rho(r_1)}{\rho(0)} \right] \delta(\mathbf{r}_1 - \mathbf{r}_2) \delta(\mathbf{r}_1 - \mathbf{r}_3) \delta(\mathbf{r}_2 - \mathbf{r}_4). \quad (1)$$

Here $C_0 = 300 \text{ MeV} \cdot \text{fm}^3$ is the inverse density of state at the Fermi surface, the standard FFS theory dimension factor for the effective interaction, and $\rho(r)$ is the density of the kind of nucleons under consideration. Typical values of the parameters (e.g. in [3]) correspond to the external constant γ^{ex} approximately ten times greater than the internal one, γ^{in} . Therefore it seems reasonable to try to find V_{eff}^p starting from the first principles, as the conditions for the validity of the Brueckner theory at the surface are much better than inside nuclei. Within the Brueckner theory, the gap equation coincides with that of the Bardeen-Cooper-Shrieffer (BCS) theory, as the ladder diagrams summation typical of the Brueckner theory is made already in the gap equation itself. In any case, the Brueckner theory is valid trivially outside the nucleus where all many-body corrections vanish, and therefore it should correctly reproduce the γ^{ex} parameter. This is not the case for the γ^{in} parameter (or, more generally, for the in-side behavior of the $V_{\text{eff}}^p(\mathbf{r}_i)$ function). But, since it is small, one can hope that even noticeable corrections to γ^{in} should not significantly change the gap Δ value. Such a logic has a weak point because of the exponential dependence of the gap on the interaction strength which is well known in the weak coupling limit of the BCS theory:

$$\Delta_F \approx 2\varepsilon_F \exp(1/\nu_F V_{\text{eff}}^p), \quad (2)$$

where $\nu_F = m^* k_F / \pi^2$ and $\varepsilon_F = k_F^2 / (2m^*)$, m^* being the effective mass. This is the reason why the knowledge of the in-side behavior of V_{eff}^p is important for accurate evaluation of the gap value, hence corrections to the BCS theory should be accounted for.

In the last few years, some progress has been made in the microscopic theory of nuclear pairing by the Milan group [6, 7] and Duguet et al. [8, 9]. And some contradictions revealed already at the “BCS level”, although both the calculations were made within rather close frameworks. In particular, the same single-particle spectrum was used for solving the gap equation, namely, it was calculated within the Skyrme–Hartree–Fock (SHF) method with the Sly4 force which produces the coordinate dependent effective mass $m^*(r)$ essentially

different from the bare one m . In Ref. [7], the value $\Delta \simeq 1.0$ MeV was found for the gap in the nucleus ^{120}Sn (a traditional benchmark for the pairing problem) which is noticeably less of the experimental one, $\Delta_{\text{exp}} \simeq 1.3$ MeV. At the same time, in [8] the value $\Delta \simeq 1.6$ MeV was obtained for the same nucleus which is essentially larger. In the first case, a lack of the gap value is explained in [7] by invoking various many-body corrections to the BCS approximation, exchange with low-lying surface vibrations (“phonons”) being the main of them. Indeed, the latter enlarges the gap value (see, e.g., [6] and [10]) making it closer to the experimental value. At the same time, it is rather difficult to find a mechanism that can reduce the value of Δ in Ref. [8]. In Refs. [11, 12] we have analyzed the reasons of these contradictions. This point was discussed also in [9]. It turned out that, in fact, these two calculations differ in the way they take into account the effective mass. It implies that the gap Δ depends not only on the value of the effective mass at the Fermi surface, as it follows from Eq. (2), but also on the behavior of the function $m^*(k)$ in a wide momentum range. But this quantity is not known sufficiently well [12] that makes rather uncertain the predictions of such calculations. To avoid it, we suggest a semi-microscopic model for the effective pairing interaction in which the main ab-initio term of V_{eff}^p is supplemented with a small addendum containing one phenomenological parameter. Preliminary results of this model were presented in [13].

2. OUTLINE OF THE FORMALISM

The general form of the many-body theory equation for the pairing gap Δ reads [5]:

$$\Delta = \mathcal{U} G G^s \Delta, \quad (3)$$

where \mathcal{U} is the NN -interaction block irreducible in the two-particle channel, and G (G^s) is the one-particle Green function without (with) pairing effects taken into account. A symbolic multiplication, as usual, denotes the integration over energy and intermediate coordinates and summation over spin variables as well. When we used above the term “BCS theory”, we meant to replace the block \mathcal{U} of irreducible interaction diagrams with the free NN -potential \mathcal{V} in Eq. (3) and to use the simple quasiparticle Green functions for G and G^s (e.g, without phonon corrections or others). In this case, Eq. (3) is greatly simplified and can be reduced

to the form usual for the Bogolyubov method,

$$\Delta = -\mathcal{V}\kappa, \quad (4)$$

where

$$\kappa = \int \frac{d\varepsilon}{2\pi i} G G^s \Delta \quad (5)$$

is the anomalous density matrix which can be expressed explicitly in terms of the Bogolyubov functions u and v ,

$$\kappa(\mathbf{r}_1, \mathbf{r}_2) = \sum_i u_i(\mathbf{r}_1) v_i(\mathbf{r}_2). \quad (6)$$

Summation in (6) is carried out over the complete set of Bogolyubov functions with eigen energies $E_i > 0$.

In Refs. [6, 7], the set of Bogolyubov equations, together with the gap equation (3) with the realistic Argonne NN -interaction v_{14} , was solved directly in the basis $\{\lambda\}$ of states restricted to the energy domain up to $E_{\max}=800$ MeV. In addition, as mentioned above, the SHF basis with the SLy4 force was used with the coordinate dependent effective mass $m^*(r)$, which is considerably smaller than the bare mass m . The main difficulty of the direct method to solve the nuclear pairing problem comes from rather slow convergence of the sums over intermediate states λ in the gap equation because of the short-range of the free NN -force. Evidently, this is the reason why the authors of [6, 7] limited the calculations only to one nucleus ^{120}Sn . To avoid the slow convergence problem, the authors of [8, 9] used the super-soft “low- k ” force $V_{\text{low-}k}$ [14] which is defined in such a way that it describes correctly the NN -scattering phase shifts at momenta $k < \Lambda$, where Λ is a parameter which is not bigger than the one corresponding to the limiting energy $E_{\text{lim}} \simeq 300$ MeV, for smaller energy values the phase shifts being reproduced accurately. As the force $V_{\text{low-}k}$ vanishes rapidly for $k > \Lambda$, one can limit the energy up to $E_{\max} \simeq 300$ MeV in the gap equation (4). This made it possible to calculate in [8] neutron and proton pairing gaps for a lot of nuclei. Usually the low- k force is found starting from some realistic NN -potential \mathcal{V} with the help of the Renormalization Group method, and the result doesn’t practically depend on the particular choice of \mathcal{V} [14]. In addition, in [8] $V_{\text{low-}k}$ was found starting from the Argonne potential v_{18} , which is different only a little from the one used in [7], v_{14} . Thus, indeed, the schemes of solving the BCS gap equation in [8] and [7] were very similar.

To overcome the slow convergence problem in the gap equation for finite systems, we used a two-step renormalization method. In this approach, we split the complete Hilbert space

of the pairing problem S to the model subspace S_0 , including the single-particle states with energies less than a fixed value of E_0 , and the subsidiary one, S' . The gap equation is solved in the model space:

$$\Delta = V_{\text{eff}}^p G G^s \Delta|_{S_0}, \quad (7)$$

with the effective pairing interaction V_{eff}^p instead of the block \mathcal{U} in the original gap equation (3). It obeys the Bethe–Goldstone type equation in the subsidiary space,

$$V_{\text{eff}}^p = \mathcal{U} + \mathcal{U} G G V_{\text{eff}}^p|_{S'}. \quad (8)$$

In this equation, the pairing effects could be neglected provided the model space is sufficiently large. That is why we replaced the Green function G^s for the superfluid system with its counterpart G for the normal system. In the BCS approximation, the block \mathcal{U} in (8) should be replaced by \mathcal{V} . To solve equation (8) in non-homogeneous systems, we have found a new form of the local approximation, the Local Potential Approximation (LPA). Originally it was developed for semi-infinite nuclear matter [15], then for the slab of nuclear matter (see review articles [2, 16]) and finally, for finite nuclei [11, 12]. It turned out that, with very high accuracy, at each value of the c.m. coordinate \mathbf{R} , in Eq. (8) the formulae of the infinite system embedded into the constant potential well $U = U(\mathbf{R})$ (it explains the term LPA) can be used. This simplifies equation for V_{eff}^p significantly, in comparison with the initial equation for Δ . As the result, the subspace S' can be chosen as large as necessary. From the comparison of the direct solution of Eq. (8) in the slab with the LPA one, it was shown that the LPA has high accuracy, even in the surface region, for sufficiently large model space, E_0 ($\simeq 20 \div 30$ MeV). For finite nuclei (the same ^{120}Sn), validity of LPA was checked also [11, 12]. In this case, the boundary energy should be made larger up to $E_0 = 40$ MeV. In this article, we use the LPA with this value of E_0 for systematic calculations of the gap in spherical nuclei. For \mathcal{V} , we use just as in [12], the Argonne potential v_{18} .

Let us note that the use of the low- k force $V_{\text{low-k}}$ could be also interpreted in terms of the two-step renormalization scheme of solving the gap equation (3), with $E_0 \simeq 300$ MeV and with free nucleon Green functions G in (8) (i.e. $U(R) = 0$). Then, (with $\mathcal{U} \rightarrow \mathcal{V}$) one obtains $V_{\text{eff}}^p \rightarrow V_{\text{low-k}}$ (see [17] where the usual renormalization scheme, similar to ours, is used to find $V_{\text{low-k}}$ instead of the Renormalization Group equation). Now, the comparison of the direct solution of the gap equation (3) (or (4)) in Ref. [7] with the Argonne NN -potential \mathcal{V} and of “renormalized” equation (7) with $V_{\text{eff}}^p = V_{\text{low-k}}$ shows that the difference

appears because, in the subsidiary subspace S' , the effective mass $m^* \neq m$ is used in the first case and $m^* = m$, in the second one. Thus, the result for the gap depends not only on the value of the effective mass at the Fermi surface, but also on the behavior of the function $m^*(k)$ in a wide momentum range. This dependence was demonstrated explicitly in [11, 12]. The use of the SHF effective mass corresponding to the SLy4 force, or to any other version of the Skyrme force, could hardly be approved. Indeed, these effective forces were introduced and fitted to describe systematically nuclear masses and radii. As a rule, the description of the single-particle spectrum nearby the Fermi surface with Skyrme forces is rather poor, and furthermore it is difficult to expect that they will reproduce it correctly at those high momenta that are involved in the gap equation (3). This point makes it problematic the problem of finding the pairing gap from the first principles completely. The situation is even more dramatic because the many-body theory equation (3) contains, in addition to the “ k -mass” of the SHF method, the “ E -mass” (inverse Z -factor) [18–20], which also is not sufficiently well known even in nuclear matter [12]. The corrections to the BCS version of Eq. (3) include also the difference of the block \mathcal{U} from the potential \mathcal{V} , mainly due to the so-called induced interaction. The attempt in [7] to find it in terms of the same SLy4 force as the nuclear mean field looks questionable. Indeed, this force was fitted to the nuclear characteristics which depend mainly on those Skyrme parameters determining the scalar Landau–Migdal (LM) amplitudes f, f' . As to the spin amplitudes g, g' , they remain practically undetermined in the SHF method. At the same time, the contribution of the spin channel to the induced interaction is not less than of the scalar one [7]. Parameters g, g' are well known from the calculations of nuclear magnetic moments within the Finite Fermi Systems (FFS) theory [21], but, just as the Skyrme parameters, at the Fermi surface only. But the states distant from the Fermi surface are important to calculate the induced interaction. At last, let us imagine to get from some phenomenology the functions $m^*(k), Z(k)$ and all the LM amplitudes far from the Fermi surface. Even in this case, the use of so many phenomenological ingredients devalues significantly the *ab initio* starting point, i.e. the free NN -potential \mathcal{V} in the pairing gap calculation.

Instead, we suggest to introduce in the effective pairing interaction a small phenomenological addendum which embodies, of course approximately, all the corrections to the BCS

scheme discussed above. The simplest ansatz for it is similar to Eq. (1) and reads:

$$\mathcal{V}_{\text{eff}} = V_{\text{eff}}^0 + \gamma C_0 \frac{\rho(r_1)}{\bar{\rho}(0)} \delta(\mathbf{r}_1 - \mathbf{r}_2) \delta(\mathbf{r}_1 - \mathbf{r}_3) \delta(\mathbf{r}_2 - \mathbf{r}_4). \quad (9)$$

Here $\rho(r)$ is the density of nucleons of the kind under consideration, and γ is a dimensionless phenomenological parameter. To avoid any influence of the shell fluctuations in the value of $\rho(0)$, $\bar{\rho}(0)$ was averaged over the interval of $r < 2$ fm. The first, *ab initio*, term in the r.h.s. of Eq. (9) is the solution of Eq. (8) with $\mathcal{U}=\mathcal{V}$ in the framework of the LPA method described above, with $m^*=m$ in the subspace S' . Then, the gap equation (7) in the model space is solved with the self-consistent basis found within the Generalized Energy Density Functional (GEDF) method [3] with the functional DF3 where the identity $m^*=m$ is assumed. The latter is of principal importance for our approach. First, it makes the results less model-dependent, all effects of $m^* \neq m$ in both model and subsidiary subspaces being attributed to the in-medium corrections beyond the pure BCS approximation. Second, single-particle spectra of the GEDF method [3] are, as a rule, in better agreement with the experimental ones than those of the popular versions of the SHF method [24]. The quality of the single-particle spectrum nearby the Fermi surface is very important for obtaining the correct value of the gap found from Eq. (4).

3. ON THE PROCEDURE TO FIND THE “EXPERIMENTAL” GAP

The gap Δ is not an observable quantity which can be extracted from experimental data directly. Usually, this quantity, Δ_{exp} , is found in terms of mass values M of neighboring nuclei via 3-term formulae,

$$2\Delta_{\text{exp}}^+(A) = \delta_2 M^+ \equiv 2M(A+1) - M(A+2) - M(A), \quad (10)$$

or

$$2\Delta_{\text{exp}}^-(A) = \delta_2 M^- \equiv 2M(A-1) - M(A-2) - M(A). \quad (11)$$

The 5-term expression is usually considered more accurate, being a half-sum of them,

$$\Delta_{\text{exp}}(A) = \overline{\delta_2 M}/2 \equiv (\delta_2 M^+ + \delta_2 M^-)/2. \quad (12)$$

These simple recipes were used, in particular, in [6–9]. However, they originate from the simplest model $\Delta = \text{const}$, and the accuracy of such prescription is not obvious *a priori*.

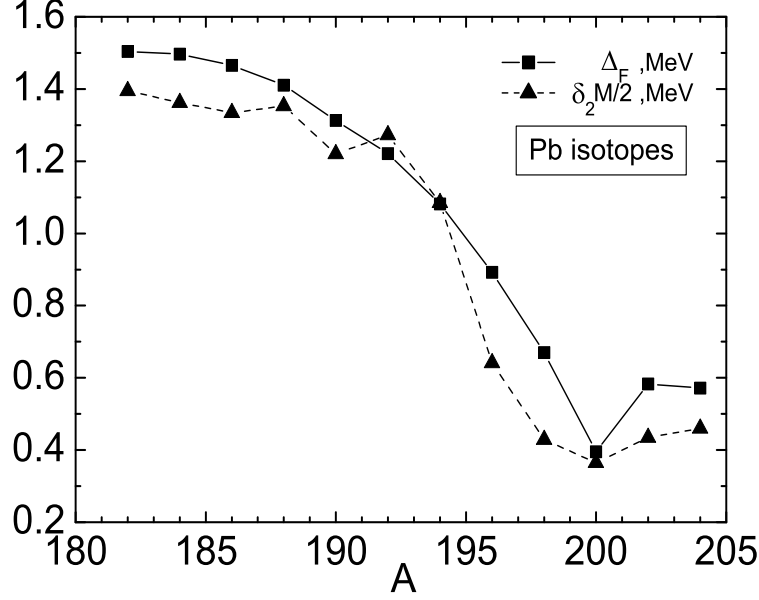


Figure 1. Comparison of the theoretical mass differences $\overline{\delta_2 M}/2$ with average gap values Δ_F for Pb isotopes

To clarify this point we made a calculation which could be considered as a “theoretical experiment”. We used the GEDF method [3] with the functional DF3 which reproduces the mass differences of Eqs. (10),(11) type sufficiently well. First, we calculated the right side of Eq. (12) directly, and second, the theoretical gap value. For the latter, we use the “Fermi average” combination,

$$\Delta_F = \sum_{\lambda} (2j+1) \Delta_{\lambda\lambda} / \sum_{\lambda} (2j+1), \quad (13)$$

where the summation is carried out over the states λ in the interval of $|\varepsilon_{\lambda} - \mu| < 3$ MeV. A similar recipe was used, e.g., in [7]. The comparison of these two quantities is given in fig. 1 for the lead isotopes and in fig. 2 for the tin isotopes. We see that for the main part of nuclei under consideration the difference between values in two neighboring columns is within 0.1 MeV. However, there is several cases where it is of the order (or even exceeds) 0.2 MeV. Leaving aside detailed analysis of these “bad” cases we are forced to put a limit of $\simeq 0.1 - 0.2$ MeV in the accuracy of the experimental gap determined from Eq. (12).

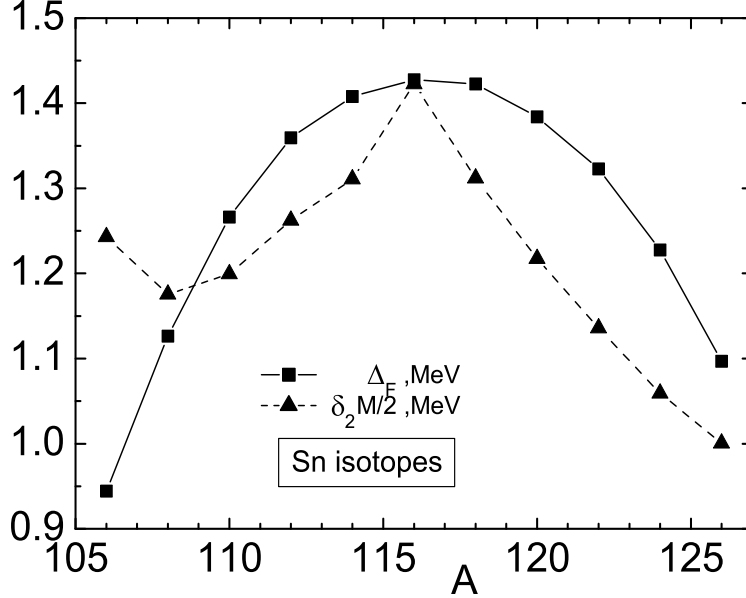


Figure 2. The same as in fig. 1, but for Sn isotopes

4. CALCULATION RESULTS

As it was discussed above, for the model space we used the GEDF method by Fayans et. al. with the DF3 functional [3]. The model space was extended up to the energy $E_0 = 40$ MeV, the subsidiary one up to $E_{\max} = 1000$ MeV. The spherical box of the radius $R = 16$ fm was used, with the grid step $h = 0.05$ fm. The numerical stability of the results was checked by increasing the parameters up to $E_0 = 60$ MeV, $E_{\max} = 1200$ MeV and $R = 24$ fm, and we found for the gap value a numerical accuracy of 0.01 MeV.

We calculated the neutron gap for 25 semi-magic isotopes of the lead, tin and calcium chains and the proton gap in 9 nuclei, also semi-magic, isotones of the $N = 82$ chain. The formulae above correspond to so-called “developed pairing” approximation [5], i.e. imposing the equality of the Δ^+ and Δ^- operators. Therefore we limit ourselves to nuclei having, as a minimum, four particles (holes) above (below) the magic core. Therefore, the only isotope ^{44}Ca was considered in the calcium chain.

Let us begin with the neutron pairing. The results are presented in table 1 and figs. 3,4. The Fermi average gap values, Eq. (13), found for different values of the parameter γ in Eq. (9). We see that the gap values with the “ab initio” interaction ($\gamma=0$) are greater by 30 – 40% than the experimental ones. This difference exceeds significantly the accuracy of $\simeq 0.1 - 0.2$ MeV for the gap value which we could expect in accordance with discussion of the

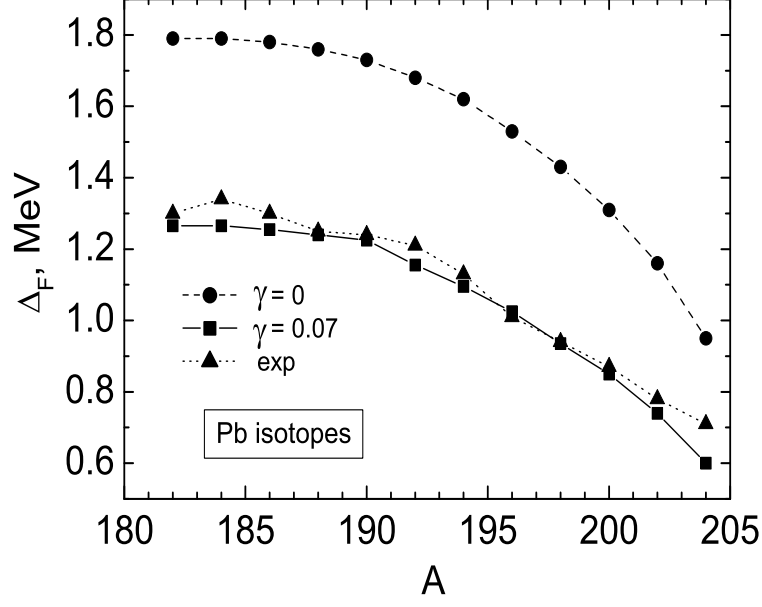


Figure 3. Neutron gap in Pb isotopes

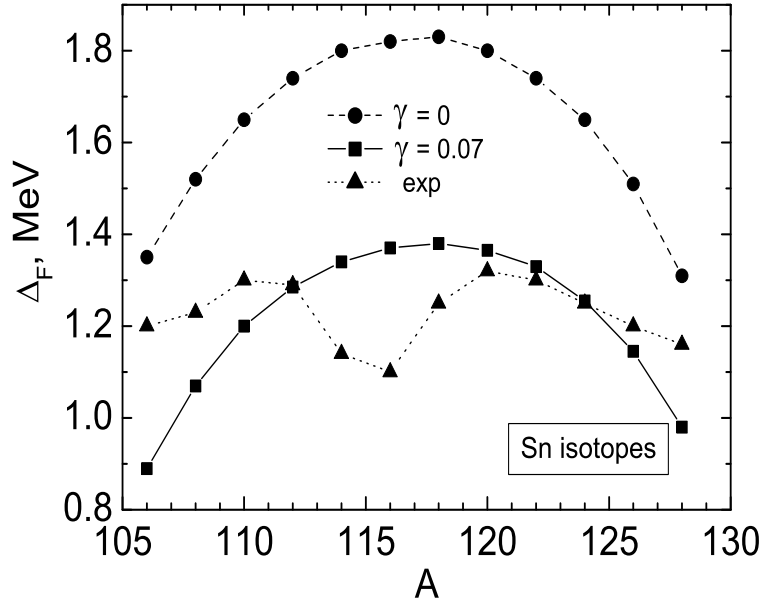


Figure 4. Neutron gap in Sn isotopes

previous section. As it can be seen, with few exceptions, it is obtained for $\gamma = 0.06 - 0.08$. For the “optimal” value of $\gamma = 0.07$ (the results are exactly half-sums of the values in the third and forth columns), the theoretical error exceeds this limit only in ^{106}Sn and ^{116}Sn . Evidently, it is caused by the fact that the DF3 functional provides an incorrect reproduction of the “intruder”-state $1h_{11/2}$, which plays an essential role in the gap equation (7) for these

Table 1. Neutron gap Δ_F^n (MeV) in semi-magic nuclei.

nucleus	Δ_F^n			Δ_{exp}
	$\gamma=0$	0.06	0.08	
^{182}Pb	1.79	1.33	1.20	1.30
^{184}Pb	1.79	1.33	1.20	1.34
^{186}Pb	1.78	1.32	1.19	1.30
^{188}Pb	1.76	1.31	1.17	1.25
^{190}Pb	1.73	1.29	1.16	1.24
^{192}Pb	1.68	1.22	1.09	1.21
^{194}Pb	1.62	1.16	1.03	1.13
^{196}Pb	1.53	1.09	0.96	1.01
^{198}Pb	1.43	1.00	0.87	0.94
^{200}Pb	1.31	0.90	0.80	0.87
^{202}Pb	1.16	0.79	0.69	0.78
^{204}Pb	0.95	0.64	0.56	0.71
^{106}Sn	1.35	0.95	0.83	1.20
^{108}Sn	1.52	1.13	1.01	1.23
^{110}Sn	1.65	1.26	1.14	1.30
^{112}Sn	1.74	1.34	1.23	1.29
^{114}Sn	1.80	1.40	1.28	1.14
^{116}Sn	1.82	1.43	1.31	1.10
^{118}Sn	1.83	1.44	1.32	1.25
^{120}Sn	1.80	1.42	1.31	1.32
^{122}Sn	1.74	1.38	1.28	1.30
^{124}Sn	1.65	1.30	1.21	1.25
^{126}Sn	1.51	1.19	1.10	1.20
^{128}Sn	1.31	1.02	0.94	1.16
^{44}Ca	1.83	1.50	1.41	1.54

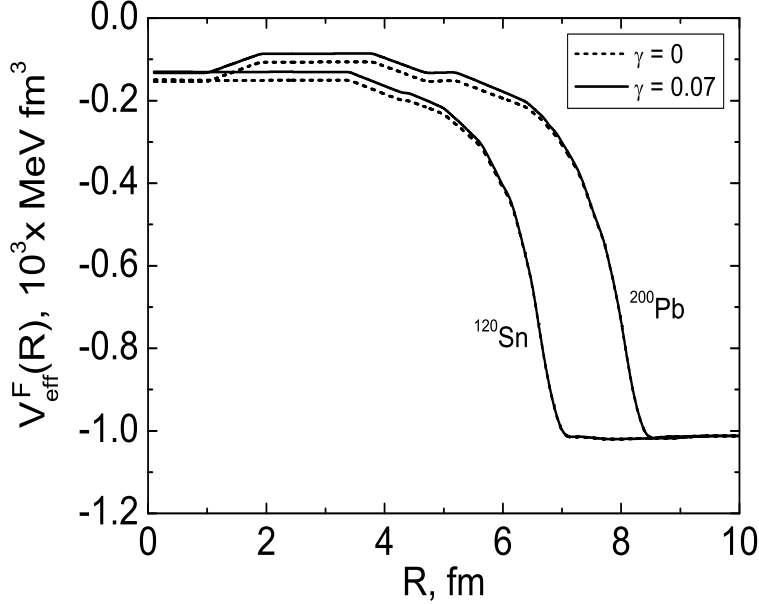


Figure 5. Fermi-average effective pairing interaction

nuclei. Fig. 3 and fig. 4 are drawn just to illustrate the optimal value of γ . To show that the phenomenological addendum to the effective pairing interaction in (9) is indeed rather small for $\gamma = 0.07$, we displayed in fig. 5 the localized “Fermi average” of the effective interaction. In the mixed coordinate-momentum representation, it is defined as follows: $\mathcal{V}_{\text{eff}}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{r}_1, \mathbf{r}_2) \rightarrow \mathcal{V}_{\text{eff}}^F(R = r_1)\delta(\mathbf{r}_1 - \mathbf{r}_2)\delta(\mathbf{r}_1 - \mathbf{r}_3)\delta(\mathbf{r}_2 - \mathbf{r}_4)$, where

$$\mathcal{V}_{\text{eff}}^F(R) = \int d^3t \mathcal{V}_{\text{eff}}(k_1 = k_2 = k_F(R), \mathbf{R} - \mathbf{t}/2, \mathbf{R} + \mathbf{t}/2), \quad (14)$$

with $k_F(R) = \sqrt{2m(\mu - U(R))}$, provided $\mu - U(R) \geq 0$, and $k_F(R) = 0$ otherwise. Here μ and $U(R)$ are the chemical potential and the potential well of the kind of nucleons under consideration. A similar quantity was considered before in the slab system to visualize the effective interaction properties [2, 25]. At a glance, the difference between the interaction strengths for $\gamma=0$ and $\gamma=0.07$ is negligible, but it produces noticeable effects in the gap due to the exponential behavior in Eq. (2).

Let us now turn to protons. In this case, the Coulomb potential \mathcal{V}_C must be added to the expression (9),

$$\mathcal{V}_{\text{eff}}^p = \mathcal{V}_{\text{eff}}^n + \mathcal{V}_C. \quad (15)$$

Again this addendum is small and again it turned out to be important for the gap equation due to the the enhancement discussed above. In particular, this was demonstrated in previous

Table 2. Proton gap Δ_F^p (MeV) for the isotone gap $N = 82$.

nucleus	Δ_F^p				Δ_{exp}
	$\mathcal{V}_{\text{eff}}^p = \mathcal{V}_{\text{eff}}^0$	$\mathcal{V}_{\text{eff}}^p = \mathcal{V}_{\text{eff}} + \mathcal{V}_C$			
		$\gamma=0$	0.06	0.08	
^{136}Xe	1.65	1.19	0.87	0.78	0.75
^{138}Ba	1.80	1.33	0.98	0.88	0.87
^{140}Ce	1.90	1.42	1.03	0.92	0.97
^{142}Nd	1.99	1.48	1.06	0.94	1.00
^{144}Sm	2.01	1.49	1.05	0.91	1.02
^{146}Gd	2.02	1.50	1.05	0.91	1.13
^{148}Dy	2.01	1.50	1.06	0.93	1.19
^{150}Er	1.98	1.48	1.07	0.94	1.22
^{152}Yb	1.92	1.44	1.05	0.93	1.29

calculations [8, 9]. The estimates show that the Coulomb potential could be taken in the bare form. Indeed, in the momentum space one has $\mathcal{V}_C = e^2/q^2$ with a strong maximum at small q values provided they persist in the matrix elements $\langle \lambda_1 \lambda_2 | \mathcal{V}_C | \lambda_3 \lambda_4 \rangle$, with obvious notation. In the gap equation, the diagonal elements with $\lambda_1 = \lambda_2 = \lambda_3 = \lambda_4$ are of primary importance for which the region around $q \simeq 0$ in the integral dominates. But at small q the Coulomb potential $\mathcal{V}_C(q)$ is not modified due to the Ward identity. In non-diagonal matrix elements the contribution of $q \simeq k_F$ dominates and $\mathcal{V}_C(q)$ could be modified, but in this case the contribution of \mathcal{V}_C is very small and can be neglected.

The results for the isotone chain $N=82$ are given in Table 2 and displayed in fig. 6. To demonstrate the effect of the Coulomb interaction, we show the results with the interaction $\mathcal{V}_{\text{eff}}^p = \mathcal{V}_{\text{eff}}^0$ which difference from the corresponding value in column 3 gives exactly the Coulomb effect in the gap. Indeed, it is rather big (about 0.5 MeV), in qualitative agreement with [8]. Again at $\gamma=0.07$ the agreement is almost perfect for the most part of nuclei, and only for the two heaviest isotones the disagreement exceeds 0.2 MeV. In this case, the possible reason lies in the proximity to the phase transition to the deformed state (at $A \simeq 150$). Average difference between the theoretical and experimental gap values for 34

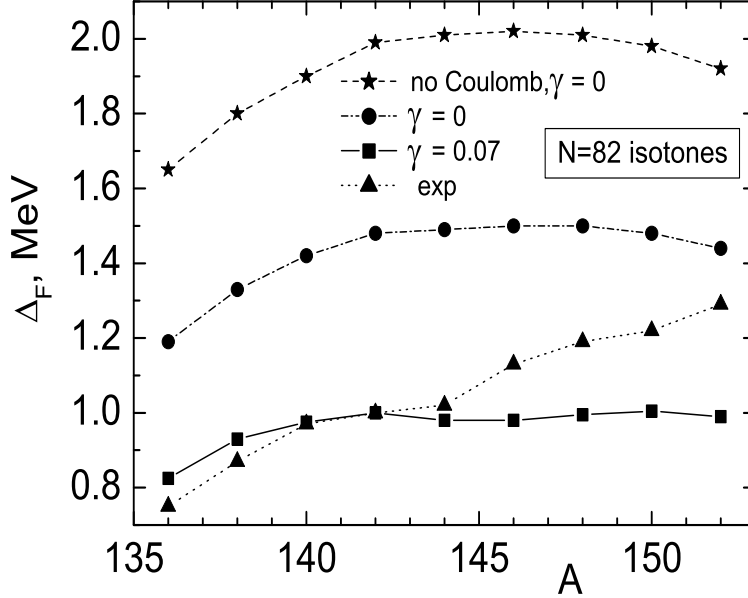


Figure 6. Proton gap in $N = 82$ isotones

nuclei considered is equal to $\sqrt{(\delta\Delta)^2} \simeq 0.13$ MeV. As it follows from the analysis in Sect. 3, this value is within the accuracy of the experimental values of the gap defined with the relation (12).

5. CONCLUSIONS

We suggest a simple semi-microscopic model (9) for the effective pairing interaction containing one phenomenological parameter which takes into account approximately various corrections to the pure BCS theory. This model reproduces rather well experimental values of the neutron and proton gaps in semi-magic nuclei. The overall agreement ($\sqrt{(\delta\Delta)^2} \simeq 0.13$ MeV) is better than that obtained in [8], where the authors did not introduce free parameters explicitly but they made it implicitly by using a specific k -dependence of the effective mass.

The ansatz of Eq. (9) possesses an obvious drawback. The phenomenological GEDF pairing interaction of [3] contains the surface term ($\propto (d\rho/dr)^2$) that plays an essential role for the description of the odd-even effect (staggering) in nuclear radii. It originates mainly from the exchange by surface phonons which was explicitly taken into account in [6, 7]. The addition of such a term in Eq. (9) is associated with introducing a new parameter, and at

the first stage we preferred to avoid it. A more consistent scheme should, evidently, include the explicit consideration of the low-lying phonons, as e.g. in [6], but taking into account the so-called tadpole diagrams [23]. In this case, the phenomenological constant γ , of course, will change.

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